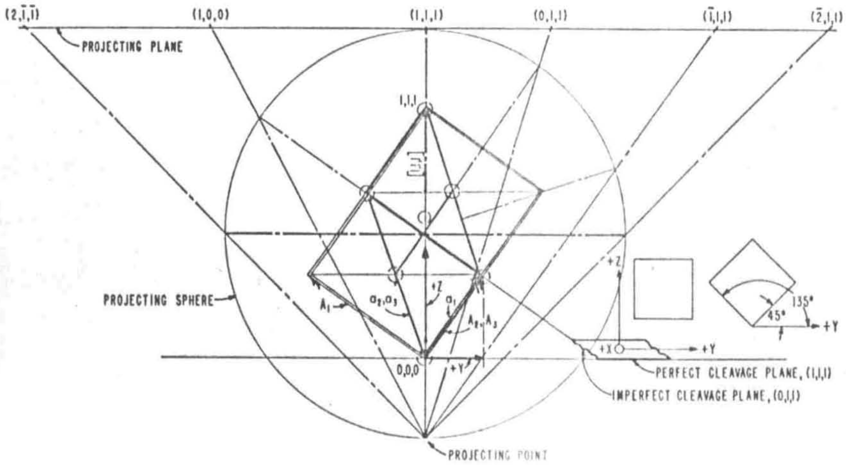

Fig. 1. Primitive and nearly cubic rhombohedral cells, stereogram of ⟨01$\bar{1}$⟩ zone (normal to the page) of the nearly cubic cell, specimen orientation and positive-sensed Cartesian axes, and position of two pronounced cleavage planes of antimony. a_1, a_2, and a_3 are the rhombohedral-cell axis vectors for the primitive cell and the outward direction of the projection of any one of them, a_1 for example, on a plane normal to the unique [111] direction is taken as $+Y$. $a_1+a_2+a_3$ is chosen as $+Z$. A_i are the cell edges of the nearly cubic cell. $+Y$ is along ($\bar{2}$,1,1) and $+X$ along [01$\bar{1}$].

are arbitrary. So that the signs of c_{14} for antimony and bismuth can be directly compared, we adopt ELR's specification for the positive-axes senses, as shown in Fig. 1.

The axes senses in the specimens were determined upon indexing a Laue diagram. (See Sec. IV.)

III. DESIGN OF EXPERIMENT

As outlined by ELR and their cited references, the six Voigt elastic stiffness constants for the class $R\bar{3}m$ are represented by

$$c_{ij}=\begin{vmatrix} c_{11} & c_{12} & c_{13} & c_{14} & 0 & 0 \\ c_{12} & c_{11} & c_{13} & -c_{14} & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ c_{14} & -c_{14} & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & c_{14} \\ 0 & 0 & 0 & 0 & c_{14} & c_{66} \end{vmatrix},$$

where $c_{66}=(c_{11}-c_{12})/2$. For acoustoelastic waves propagating with direction cosines l, m, n, in this order relative to the X, Y, and Z axes of a right-handed coordinate system, three values for the velocities (one longitudinal and two transverse) satisfy the Christoffel determinant. Symmetry, however, prevents one from choosing six of the nine possible modes which would allow the direct (and accurate) determination of the six constants on one simply shaped oriented specimen. Consequently, it is necessary to employ a minimum of two differently oriented single-crystal cubes and more than the minimum of six modes required in principle to determine six constants. All but c_{13} are best arrived at when (1) they derive from velocities of three modes propagating along each coordinate-axis direction on one specimen and (2) the velocity data so obtained are self-consistent. Accordingly, one of the two specimens needed is a cube with faces normal to the principal axes. To determine c_{13} symmetry requires one to employ a mode propagating at any angle with the trigonal axis other than 0°, 90°, and 180° (plus four of the five previously discussed constants). Two directions (45° and 135° with the Y axis in the Y-Z mirror plane) exist for which c_{13} makes its maximum contribution to the effective stiffness constant, and our second cube is oriented with faces normal to these directions and normal to the X direction. To insure that the five already determined values for the constants are the same for this cube, velocity data are obtained for its nine possible modes.

In all, 18 velocity measurements are required. Because one of them corresponds to a doubly degenerate shear mode along the Z axis, and three others repeat the modes along the X axis on the second orientation, only 14 velocities need be analyzed in detail. Clearly, these must satisfy 8 redundancy relations for a meaningful calculation of the six elastic constants. The 14 expressions for the effective stiffness constants ρv_i^2 are listed in Table I. (The symbols v_1 through v_{14} are chosen to correspond to ELR's arbitrary assignment.) Also included are the wave-propagation and transducer-polarization-direction cosines, the numerical values of the averaged observed velocities, and the experimental tolerances.

IV. EXPERIMENTAL DETAIL

The velocity of sound was determined by the ultrasonic pulse-echo method. A pulse width of approximately 2 μsec wide was used and the distance between the maximum amplitude of successive unrectified radiofrequency pulses was used as a measure of the transit time.[8] Transit-time error effects were also investigated by means of the dummy-transducer method.[9] Times were measured on a Tektronix 585A oscilloscope whose timing circuit was checked with a counter (Hewlett-Packard 524B) and a quartz signal generator (Tektronix Time Marker Generator 180-S1). An Arenberg PG-65-C pulse generator, preamplifier PA-620-B, and wideband amplifier WA-600-B were used to generate and amplify the pulses. An X- or Y-cut-quartz transducer of 10- or 5-Mc/sec fundamental frequency func-

[8] S. Eros and J. R. Reitz, J. Appl. Phys. 29, 683 (1958).
[9] C. S. Smith and J. W. Burns, J. Appl. Phys. 24, 15 (1953).